## Complex Systems 511, Fall 2013: Theory of Complex Systems

Time: Monday, 2-5 p.m.
Room: 1210 Weill Hall
Instructor: Seth Marvel
Office: 4851 East Hall
Office hours: Tuesday, 9:30-11:30 a.m. and by appointment
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### COURSE DESCRIPTION

This course provides an introduction to the mathematical and philosophical foundations of the study of complex systems. We begin with a survey of deterministic nonlinear dynamics and chaos. This provides a concrete context for understanding the meaning of terms like stability, robustness, universality, criticality, and emergence. We then examine a series of nondeterministic models that illustrate some of the techniques used to analyze stochastic complex systems. These examples also serve to highlight themes of best practice for research in the field. We conclude with several additional topics, including information theory, networks, and big ideas that have inspired the discipline. Our tour will consider examples from population dynamics, microbiology, evolutionary theory, epidemiology, condensed matter physics, opinion formation, and the environment.

### REQUIREMENTS

A firm command of calculus and linear algebra is a requirement. In particular, students should be comfortable solving linear ordinary differential equations and with the calculation and properties of eigenvalues and eigenvectors of matrices. Some experience with computer programming (e.g. in Matlab or Mathematica) would also be helpful.

### COURSEWORK

There will be biweekly graded problem sets, one take-home midterm and a significant final project to be done individually. Collaboration is allowed for the problem sets, but the take-home exam must be done alone. The exam will be given out at the end of class on Oct. 28 and will be due at the beginning of class on Nov. 4. Grades will be calculated from the problem sets (30%), midterm (30%), final project (30%), and in class participation during the term (10%).

### TEXT

The content of the course is broad and no text on the market adequately covers it. So there is no required text. However, the first portion of the course (the first three classes) will draw directly out of *Nonlinear Dynamics and Chaos*, by Steven Strogatz (Westview Press, 2001). I recommend acquiring a copy, though it remains optional. Additional reading assignments will be posted on the course website and simultaneously added to a class M+Box folder at least one week before we discuss them in class. You are expected to peruse these and be able to talk about them in class.

**COURSE OUTLINE** (Tentative. Titles, topics, and problem set dates may change.)

**Sept. 9: Overview and introduction to the study of complex systems**
(Topics: modeling as local to global behavior, quantitative vs. qualitative accuracy in modeling, dynamics vs. statistics, applications of physics to nonphysical questions)

***Intuitions from Deterministic Dynamics***

**Sept. 9: Limit sets, stability, and robustness**
(Topics: fixed points and other limit sets, stability, basins of attraction, existence and uniqueness, robustness and universality)
*Reading: Nonlinear Dynamics and Chaos. Ch. 1-2, 5-6.*
Problem Set 1 assigned.

**Sept. 16: Oscillations, bifurcations, and phase transitions**
(Topics: oscillations, limit cycles, potentials, path dependence and independence, Lyapunov functions, reversible and irreversible systems, conservative and nonconservative systems, bifurcations, phase transitions, coupled oscillator systems and symmetry breaking, reaction-diffusion equations)
*Reading: Nonlinear Dynamics and Chaos. Ch. 3-4, 7-8.*

**Sept. 23: Chaos, iterated maps, and prediction**
(Topics: strange attractors, chaos, exponential divergence of chaotic trajectories, fractals, the logistic and Hénon maps, fractal dimension)
*Reading: Nonlinear Dynamics and Chaos. Ch. 9-12.*
Problem Set 1 due.
Problem Set 2 assigned.

***Models and Concepts from Stochastic Dynamics***

**Sept. 30: The random walk and stable distributions**
(Topics: random walks, central limit theorem, alpha stable distributions, heavy-tailed random walks, first passage times, generating function approaches)

**Oct. 7: Power laws and similar distributions**
(Topics: power laws, scaling laws in biology and urban metrics, Benford's law, Zipf's law, the Pareto distribution, branching processes, the Yule process and preferential attachment)
Problem Set 2 due.
Problem Set 3 assigned.

**Oct. 21: Criticality**
(Topics: percolation, Erdös-Rényi random graphs, self-organized criticality, the Ising model, critical exponents, renormalization)
Problem Set 3 due.
Midterm 1 handed out.

**Oct. 28: Coarsening**
(Topics: Schelling's model of segregation, explosive percolation, Kingman's coalition, spinodal decomposition, Ostwald ripening, Axelrod's model of cultural dynamics)
Midterm 1 due.
Problem Set 4 assigned.

**Nov. 4: Random sequential absorption**
(Topics: recurrence relations, random sequential absorption, randomized greedy matching, jamming problems)

**Nov. 11: Opinion dynamics**
(Topics: spin glasses, structural balance, frustration, Markov processes, landscape models, metastability, simulated annealing, Master equation formulations)
Problem Set 4 due.
Problem Set 5 assigned.

***Additional Topics***

**Nov. 18: Cellular automata and entropy**
(Topics: cellular automata, measures of information and entropy, Turing machines, polysemy in science, complex adaptive systems)

**Nov. 25: Uniformly random structures**
(Topics: Poisson arrival, Poisson point processes, geometric random graphs, random matrix theory, Wigner's semicircle law, other types of universality in random matrix theory)
Problem Set 5 due.
Problem Set 6 assigned.

**Dec. 2: Networks and network analysis**
(Topics: network science, centrality measures, modularity and community detection, complex network classification)

**Dec. 9: Toward answers to big questions**
(Topics: origin of life, major transitions, genetic algorithms, strategy and cooperation, More is Different, schema theory, the "edge of chaos," Kolmogorov complexity)
Problem Set 6 due.