Review Worksheet
M, Apr. 22

1. Vector fields, gradients, and work

   (a) What is a conservative vector field?

   (b) What do we know about line integrals along curves lying within a conservative vector field?

   (c) Suppose we are given a function

   $$z = g(x, y) = 4 - x^2 - y^2.$$

   Let $f(x, y, z) = z - g(x, y) = 0$. In the same sketch, draw $g(x, y)$, ∇f, and ∇g.

   (d) How is work defined? What do we know about the work done by a conservative vector field in moving a particle along a path C? Why?

   (e) Find the work done by the force field

   $$\mathbf{F}(x, y, z) = \left\langle \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \right\rangle$$

   in moving a particle along the straight line segment from $(1, 1, 1)$ to $(2, 2, 2)$.

2. Arc length and surface area

   (a) Broadly, what is our default strategy for computing arc length and surface area (also the default strategy for computing line and surface integrals)?

   (b) Find the area of the parametric surface $\mathbf{r}(u, v) = \langle u^2, uv, \frac{1}{2}v^2 \rangle$ on the rectangle $0 \leq u \leq 1$, $0 \leq v \leq 2$.

3. Divergence and curl

   (a) Roughly, what geometrically do divergence and curl represent?

   (b) Let $\mathbf{r} = \langle x, y, z \rangle$ and

   $$\mathbf{F} = \frac{\mathbf{r}}{|\mathbf{r}|^3}.$$

   Compute $\nabla \cdot \mathbf{F}$ for all $r \neq 0$.