Department of Mathematics
University of Michigan

Math 255 Applied Honors Calculus III
April 26, 2013

Final Exam

Closed book and notes. No calculators.
YOU MUST SHOW YOUR WORK IN FULL DETAIL.
A scratch sheet is included as the last page of the exam.

Name:______________________________ **Section:**______

Question	Points	Score
1	25	
2	25	
3	25	
4	25	
5	25	
6	25	
7	25	
8	25	
Bonus 1	10	
Bonus 2	10	
Total	200	

1. (25 points) What is the acute angle between two faces of a regular tetrahedron? (Hint: The tetrahedron may be rescaled and repositioned in $\mathbb{R}^3$ so that its vertices, or corners, are located at the corners of the box $(1,1,1)$, $(1,-1,-1)$, $(-1,1,-1)$, and $(-1,-1,1)$.)

2. (25 points) Find all $\langle x, y, z \rangle$ on the curve $\mathbf{r}(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle$ at which the osculating plane is parallel to the plane $2x + 2y + z = 2$.

3. (25 points) Consider the two lines:

$$\begin{aligned} x_1 &= s - 1, \\ y_1 &= s, \\ z_1 &= 0, \end{aligned} \qquad \text{and} \qquad \begin{aligned} x_2 &= 2t - 1, \\ y_2 &= 0, \\ z_2 &= t - 1. \end{aligned}$$

(a) The distance between a point (x_1, y_1, z_1) on the first line and a point (x_2, y_2, z_2) on the second line is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Find the minimum distance between the two lines by finding the critical points of a suitable two-variable function. (You may assume that one of the critical points you find corresponds to the minimum distance.)

(b) Using the result from part (a), determine whether the lines are parallel, intersecting or skew. Justify your answer.

4. (25 points) Use a suitable function $f(x, y, z)$ and a linear approximation at a suitable point (x_0, y_0, z_0) to approximate

$$\frac{(4.98)^2}{\sqrt{(3.01)^2 + (3.95)^2}}.$$

5. (25 points) Find the work done by the force field $\mathbf{F}(x, y) = (1 + xy)e^{xy}\,\mathbf{i} + x^2 e^{xy}\,\mathbf{j}$ in moving a particle along the curve $\mathbf{r}(t) = \langle t + \sin(\pi t), t + \cos(\pi t)\rangle$, $0 \leq t \leq 1$.

6. (25 points) Find the area of the parametric surface $\mathbf{r}(u,v) = \langle 1 - u^2 - v^2, -v, -u \rangle$, $u^2 + v^2 \leq 2$.

7. (25 points) Compute $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + (x^2 + y^2 + z^2)\,\mathbf{k}$ and S is the part of the surface

$$h(x, y) = \frac{\sin(x^2 + y^2)}{x^2 + y^2}$$

that lies within the cylinder $x^2 + y^2 = 4\pi^2$. S is oriented upward.

8. (25 points) Let $\mathbf{F}(x, y) = \langle x + e^{y^2+z^2}, \cos(x^2 z^2) + y, \sin(xy) + z\rangle$, and let S be the part of the half-cone $x = \sqrt{y^2 + z^2}$ that lies between the planes $x = 0$ and $x = 2$, oriented in the direction of the positive x-axis. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

Bonus Problem 1
(10 extra points, up to an exam total of 200 points)

9. Let

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}.$$

(a) Find $f_x(0,0)$ and $f_y(0,0)$.

(b) Is f differentiable at $(0,0)$?

Bonus Problem 2
(10 extra points, up to an exam total of 200 points)

10. Use the Divergence Theorem to evaluate

$$\iint_S (x^2 - 2y + z)\, dS,$$

where S is the sphere $x^2 + y^2 + z^2 = 1$.

Scratch Sheet