MATH 255 Applied Honors Calculus III Winter 2009

Lab 2: Quadric Surfaces and Parametric Space Curves with Maple

The objective of this lab is to familiarize yourself with the shapes and properties of cylinders, quadric surfaces (section 13.6), and parametric space curves (section 14.1).

Preliminaries.

Start Maple by clicking on the icon that appears either on your desktop or the dock. If asked, select to open a "worksheet" rather than a "document". Go to the **Maple** menu and select **Preferences**. A dialog window will pop up, click on **Display** and select **window** for **Plot display**, then click the **Apply to Session** button at the bottom of the box (this will make each plot appear on a separate window). If necessary, to open a new worksheet, go to the **File** menu and select **New** and choose new worksheet.

At the prompt, type:

```
> with(plots):
```

This will allow you to use different plotting commands available in Maple. *Note: don't forget to type a semicolon (`;`) and hit the return key at the end of each command. You can also end a command with a colon (`:`) if you don't want to see the output.*

Plotting cylinders and quadric surfaces.

To plot cylinders and quadratic surfaces in Maple we can use the `implicitplot3d` command. The syntax for this command is (no need to type this):

> implicitplot3d(*expression*, x=a..b, y=c..d, z=e..f, *options*);

where *expression* is some expression involving `x`, `y`, and `z`, and `a`, `b`, `c`, `d`, `e`, and `f` are real constants specifying the ranges of `x`, `y`, and `z`. For example, a cylinder can be plotted using:

```
> implicitplot3d(x^2 + y^2 = 1, x=-1..1, y=-1..1, z=-3..3);
> implicitplot3d(x^2 + z^2 = 1, x=-1..1, y=-3..3, z=-1..1);
> implicitplot3d(z = sin(x), x=-2*Pi..2*Pi, y=-6..6, z=-2..2);
```

Now try now some of the quadric surfaces described in the table at the end of section 13.6 (pg. 872). There are three types you should get familiar with and be able to recognize and sketch (roughly):

- Ellipsoids.

  ```
  > implicitplot3d((1/4)*x^2 + (1/9)*y^2 + (1/16)*z^2 = 1,
      x=-2..2, y=-3..3, z=-4..4);
  ```

- Elliptic Paraboloids.

```
> implicitplot3d(z=(1/4)*x^2 + (1/9)*y^2,
      x=-4..4, y=-6..6, z=0..6, scaling=constrained);
> implicitplot3d(y=(1/4)*x^2 + (1/9)*z^2,
      x=-4..4, y=0..6, z=-6..6, scaling=constrained);
> implicitplot3d(x=4*(y-2)^2 + (z-2)^2,
      x=0..10, y=-1..5, z=-2..6);
```

- Cones. These are special cases of hyperboloids (see below) where the constant term in the equation for the surface is zero.

```
> implicitplot3d(z^2=(1/4)*x^2 + (1/9)*y^2,
      x=-3..3, y=-4..4, z=-1..1);
> implicitplot3d(y^2=(1/4)*x^2 + (1/9)*z^2,
      x=-3..3, y=-1..1, z=-4..4);
```

Other quadric surfaces that we'll come across are hyperboloids. Although they are defined by similar equations, there are two types: hyperboloids of one sheet are connected surfaces:

```
> implicitplot3d((1/4)*x^2 + (1/9)*y^2 - z^2 =1,
      x=-5..5, y=-7..7, z=-2..2);
> implicitplot3d(-x^2 + (1/9)*y^2 + (1/4)*z^2 =1,
      x=-2..2, y=-7..7, z=-5..5);
```

while hyperboloids of two sheets are really two disjoint surfaces represented by a single equation:

```
> implicitplot3d((1/4)*x^2 - (1/9)*y^2 - z^2 =1,
      x=-4..4, y=-6..6, z=-2..2);
```

Challenge question: one way to think of a cone is as a hyperboloid that is neither of one sheet nor of two sheets, but is rather on the boundary between these two cases. Come up with a Maple animation to illustrate this point of view. What should the parameter indexing the frames of the animation be?

Later in the course we will also see hyperbolic paraboloids:

```
> implicitplot3d(z = - (1/4)*x^2 + (1/9)*y^2,
      x=-5..5, y=-6..6, z=-2..2);
```

Make some plots of a few hyperbolic paraboloids until you see why are sometimes called "saddle surfaces".

Plotting in cylindrical and spherical coordinates.

Maple can also plot surfaces expressed in cylindrical and spherical coordinates using the `plot3d` command:

```
> plot3d(r(theta,z), theta=a..b, z=c..d, coords=cylindrical);
> plot3d(rho(theta,phi), theta=a..b, phi=c..d, coords=spherical);
```

Here are some examples:

- Plots in cylindrical coordinates.

  ```
  > plot3d(1, theta=0..2*Pi, z=-3..3, coords=cylindrical);
  > plot3d(z, theta=0..2*Pi, z=0..3, coords=cylindrical);
  ```

- Plots in spherical coordinates.

  ```
  > plot3d(1, theta=0..2*Pi, phi=0..Pi, coords=spherical);
  > plot3d(sin(theta)*cos(phi), theta=0..2*Pi, phi=0..Pi, coords=spherical);
  ```

Plotting parametric curves in 3D.

The Maple command `spacecurve` allows you to plot parametric curves in space. Try, for instance, the parametric equations of a line:

```
> spacecurve([1+ t, 2 - 3*t, -1 - t, t=0..10]);
```

Here are some other examples to try:

```
> spacecurve([cos(5*t), sin(5*t), t, t=0..Pi]);
> spacecurve([cos(5*t), sin(5*t), log(t), t=0.1..Pi]);
> spacecurve([t, 1/(1+t^2), t^2, t=-10..10]);
> spacecurve([cos(t), sin(t), sin(5*t), t=0..2*Pi]);
```

Sometimes it is hard to see the three-dimensional character of a space curve, and in this case the `tubeplot` command is also useful; instead of just drawing the space curve, it draws a little tube around the curve. Try this one:

```
> tubeplot([cos(5*t), sin(5*t), log(t)], t=0.1..Pi, radius=0.1);
```

Try comparing plots generated by `spacecurve` and `tubeplot` for various parametric equations. You may need to adjust the radius parameter to make the tube the right thickness.

That's all! Be sure to hang on to this lab manual as it may be helpful in using Maple to make plots for homework later on.