Worksheet 1
F, Jan. 18

1. *Collinearity Test*
   (a) Given three points P, Q, R, can you use vectors to determine whether all these points lie on one line?
   (b) Stewart 12.1.9.
   (c) On a related note, three non-collinear points P, Q, R determine a unique plane. Given the points, how can you find the normal vector $\mathbf{n}$?
   This suggests another way to test collinearity. Which test is computationally easier?
   (d) Stewart 12.5.33.

2. *Coplanarity Test*
   (a) Given four points P, Q, R, S, can you use vectors to determine whether all these points lie on one plane?
   (Hint: You can use vectors $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{PS}$.)
   (b) Stewart 12.4.37.

3. Let $\mathbf{a}, \mathbf{b}$ be the position vectors of two fixed points A, B. Consider the vector equation $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$.
   (a) If the position vector $\mathbf{r}$ of a point P satisfies this equation, what does that say about $\angle APB$?
   (b) In $\mathbb{R}^2$, what is the set of all points P that satisfies this condition? Can you guess what curve/surface would satisfy the same condition in $\mathbb{R}^3$?
   (c) The given vector equation can be rewritten in the form $\mathbf{r} \cdot \mathbf{r} - 2\mathbf{c} \cdot \mathbf{r} + \mathbf{c} \cdot \mathbf{c} = \mathbf{d} \cdot \mathbf{d}$. Find vectors $\mathbf{c}$ and $\mathbf{d}$. Use this to justify your guess in part (b) and to describe that curve/surface.

4. Stewart 12.3.55.

5. Stewart 12.4.44.