Department of Mathematics
University of Michigan

Math 255 Applied Honors Calculus III
February 19, 2013

Midterm 1

Closed book and notes. No calculators.
YOU MUST SHOW YOUR WORK IN FULL DETAIL.
A scratch sheet is included as the last page of the exam.

Name:______________________________ **Section:**______

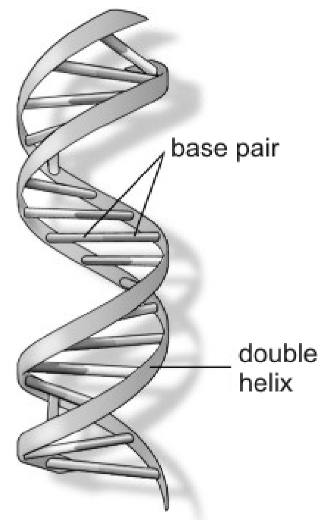
Question	Max.	Score
1	20	
2	15	
3	20	
4	20	
5	5	
6	20	
Bonus	10	
Total	100	

1. (20 points) Are the lines

$$\frac{x}{2} = \frac{y+2}{3} = z \quad \text{and} \quad x - 3 = \frac{y-1}{2} = \frac{z+2}{2}$$

parallel, intersecting, or skew? If they intersect, find the point of intersection. If they are parallel or skew, find the distance between them.

2. (15 points) A new form of life is discovered. It has a DNA-like molecule in the shape of a double helix. The radius of each helix is 6.4 Å or about $40/(2\pi)$ Å. Each helix rises about 30 Å during each complete turn, and there are about 2×10^8 complete turns in the full molecule. The length of the helix between adjacent base pairs (see the figure below) is about 5 Å. By an arc length calculation, find the total number of base pairs in the molecule.



3. (20 points)

(a) Find the equation of the tangent plane to the top half of the ellipsoid $x^2 + 4y^2 + z^2 = 6$ at the point $(1, -1, 1)$.

(b) Find the equation of the tangent plane to the cylinder $x^2 + y^2 = 2$ at the same point, $(1, -1, 1)$.

(c) At the point $(1, -1, 1)$, find the equation of the normal plane to the curve of intersection of the semi-ellipsoid from part (a) and the cylinder from part (b). (Recommendation: Use the tangent planes from parts (a) and (b) to obtain your solution. What observation justifies doing so?)

4. (20 points) Let

$$f(x,y) = \begin{cases} \dfrac{x^2 \sin y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) Is $f(x,y)$ continuous? Justify your answer.

(b) Find the directional derivative $D_{\mathbf{u}}f(0,0)$, if it exists, for each unit vector $\mathbf{u} = \langle a, b \rangle$.

(c) Is f differentiable everywhere on $\mathbb{R}^2$? Explain.

5. (5 points) Let $\mathbf{T}$ be the unit tangent vector of a parametric curve C. (Assume $\mathbf{T}$ exists at every point on C.) Show that $\mathbf{T} \cdot \mathbf{T}'' \leq 0$.

6. (20 points) Let $g(s,t) = f(x(s,t), y(s,t))$, where

$$x(s,t) = \frac{1}{s - t^2} \quad \text{and} \quad y(s,t) = \ln(s - t).$$

Use the chain rule and a linear approximation to estimate the value of $g(1.9, 1.05)$ given that $f(1,0) = 5$ and $\nabla f(1,0) = (3, -1)$.

Bonus Problem
(10 extra points, up to an exam total of 100 points)

7. Let $\mathbf{r} = \langle x, y, z \rangle$ and let $\mathbf{u} = \langle a, b, c \rangle$ be a unit vector. The function $f(\mathbf{r}) = |\mathbf{r}|$ is differentiable everywhere on $\mathbb{R}^3$ except $\mathbf{r} = \mathbf{0}$. Compute $D_{\mathbf{u}} f$ and express the result in terms of $\mathbf{r} \cdot \mathbf{u}$ and $|\mathbf{r}|$ only.

Scratch Sheet