Review Worksheet

F, Mar. 22

1. Critical points of $f(x,y)$.
   (a) How do we find the maxima, minima, and saddle points of f?
   (b) Find the local maxima, minima, and saddle points of
   $$f(x,y) = e^y(y^2 - x^2).$$
2. Extrema of $f(x,y)$ on a constraint $g(x,y) = c$.
   (a) What two ways have we learned for finding the maximum and minimum values of $f(x,y)$ on the constraint $g(x,y) = c$?
   (b) Three alleles (alternative versions of a gene) A, B, and O determine the four blood types A (AA or AO), B (BB or BO), O (OO), and AB. Under Hardy-Weinberg conditions (large population size, random mating, no mutations, no natural selection, no migration, and no meiotic drive), the proportion of individuals in a population that carry two different alleles is given by
   $$D = 2pq + 2pr + 2qr,$$
   where p, q, and r are the proportions of A, B, and O in the population. Use the fact that $p + q + r = 1$ to show that D is at most $\frac{2}{3}$.
   (c) Find the maxima and minima of $f(x,y) = y(y-2)$ on the constraint $g(x,y) = x^4 + y^4 = 16$.
3. Iterated integrals and Fubini's Theorem
   (a) Under what conditions can we reverse the order of integration in a double integral without changing the limits of integration?
   (b) Using a change of variables, we can verify that
   $$\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\, dy\, dx = \frac{1}{2} \qquad \text{but} \qquad \int_0^1 \int_0^1 \frac{x-y}{(x+y)^3}\, dx\, dy = -\frac{1}{2}.$$
   How can we reconcile this with Fubini's theorem?