

Review Worksheet

M, Mar. 25

1. Multiple integrals over general regions.

(a) What are the two general cases in which we need to rewrite a double integral before calculating its value?

(b) Evaluate:

$$\int_0^1 \int_y^1 x^{-1} e^{-y/x} dx dy.$$

(c) S is the solid bounded by the planes $x = 0$, $x + y = 2$, and $z = -(y - 1)/2$, and the circular cylinder $y^2 + z^2 = 1$. Set up an iterated integral for the volume of S . Do not evaluate.

2. Polar and cylindrical coordinates.

(a) Broadly, what are the two general cases in which we might want to make a change of variables to polar or cylindrical coordinates?

(b) Evaluate:

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \frac{1+x}{(1+x)^2 + y^2} dy dx.$$

(c) Liquid parabolic mirrors are used pervasively in astronomy. To make such a mirror, a shallow round pan containing mercury is rotated, causing the surface of the mercury to assume the shape of a circular paraboloid (see board). Suppose as chief senior scientist of a company that specializes in high end, made-to-order telescopes, you want to build a mirror with radius R and paraboloid

$$z = \epsilon(x^2 + y^2),$$

where ϵ is a known constant. Compute the minimum amount of mercury needed.