

Review Worksheet

Tu, Mar. 26

1. A general point about multiple integrals.

- (a) Suppose we want to calculate the volume of a solid in some coordinate system. Should this volume be expressed as a double integral with an integrand expressing the change in height, or a triple integral with 1 as the integrand?
- (b) Liquid parabolic mirrors are used pervasively in astronomy. To make such a mirror, a shallow round pan containing mercury is rotated, causing the surface of the mercury to assume the shape of a circular paraboloid (see board). Suppose as chief senior scientist of a company that specializes in high end, made-to-order telescopes, you want to build a mirror with radius R and paraboloid

$$z = \epsilon(x^2 + y^2),$$

where ϵ is a known constant. Determine the minimum amount of mercury needed.

2. Spherical coordinates.

- (a) Broadly, what are the two cases in which we might want to make a change of variables to spherical coordinates?
- (b) Derive an equation in spherical coordinates for each of the following:
 - i. circular upper half-cone,
 - ii. circular cylinder,
 - iii. the circular hyperboloid of two sheets, $z^2 - x^2 - y^2 = c^2$.
- (c) Using spherical coordinates, find the volume of the solid inside both the circular cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$ and above the xy -plane.

3. Change of variables in multiple integrals.

- (a) What is one feature of a given problem that makes it reasonable to look for a change of variables?
- (b) Review 15.10.14. (I did this problem in class on W, Mar. 20.)
- (c) Use the transformation, $x = u/v$ and $y = v$, to evaluate $\iint_D xy dA$, where D is the region in the first quadrant bounded by the lines $y = x$ and $y = 2x$ and the hyperbolas $xy = 1$ and $xy = 4$.