

## Midterm 2

Closed book and notes. No calculators.  
YOU MUST SHOW YOUR WORK IN FULL DETAIL.  
A scratch sheet is included as the last page of the exam.

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Question	Max.	Score
1	20	
2	15	
3	10	
4	15	
5	20	
6	20	
Bonus	10	
<b>Total</b>	100	

1. (20 points) Use Lagrange multipliers to solve for all  $(x, y)$  pairs at which

$$f(x, y) = x(x - 2) + y(y - 2) + 2xy$$

may have global (absolute) maxima or minima on

$$g(x, y) = x^2 + y^2 = 2.$$

You do not need to evaluate  $f$  at these points or determine the extreme values of  $f$  on  $g = 2$ .

2. (15 points) Evaluate the integral,

$$\int_0^\pi \int_{x/\pi}^1 y^2 \cos(xy) dy dx.$$

3. (10 points) Evaluate the integral,

$$\iint_D \frac{y}{x^2 + y^2} dA,$$

where  $D$  is the region in the first quadrant enclosed by the circles  $x^2 + y^2 = 2$  and  $x^2 + y^2 = 4$  and the lines  $y = x$  and  $x = 0$ .

4. (15 points)  $S$  is the solid bounded by the parabolic cylinder  $y = z^2$  and the two planes  $x + y + z = 2$  and  $x = 0$ . Set up an iterated integral for the volume of  $S$ . You do not need to evaluate this integral.

5. (20 points) Use spherical coordinates to evaluate the integral,

$$\iiint_E \frac{dV}{x^2 + y^2 + z^2},$$

where  $E$  is the region inside the sphere  $x^2 + y^2 + z^2 = 2$  and outside the cylinder  $x^2 + y^2 = 1$ .

6. (20 points) Let  $D$  be the region in the first quadrant bounded by the lines  $y = x$  and  $y = 3x$  and the hyperbolas  $y = 1/x$  and  $y = 3/x$ . Find equations for a transformation,  $x = f(u, v)$  and  $y = g(u, v)$ , that maps a rectangular region  $S$  in the  $uv$ -plane onto  $D$ . Use this transformation to evaluate the integral  $\iint_D xy \, dx \, dy$ .

**Bonus Problem**

(10 extra points, up to an exam total of 100 points)

7. Set up an iterated integral using cylindrical coordinates to find the volume of the solid bounded by the circular paraboloid  $z = x^2 + y^2$  and the plane  $z = y$ . You do not need to evaluate this integral.

## Scratch Sheet