Problem Set 1 – Solutions

12.1.20 Center of sphere C is midpt btwn endpts:

$$C = \left(\frac{2+4}{2}, \frac{1+3}{2}, \frac{4+10}{2}\right) = (3,2,7)$$

Radius of sphere R is half dist. btwn endpts:

$$R = \frac{1}{2}\sqrt{(2-4)^2+(1-3)^2+(4-10)^2} = \sqrt{11}$$

So eq. of sphere is

$$\boxed{(x-3)^2+(y-2)^2+(z-7)^2 = 11}$$

12.2.8 $\vec{u}+\vec{v}+\vec{w}=0$, so $\vec{u},\vec{v},\vec{w}$ form a triangle:

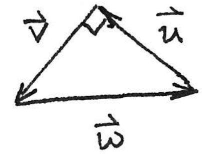
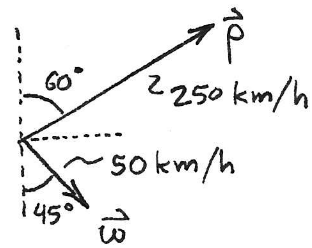


Fig. in book shows $\vec{u}$ & $\vec{v}$ as $\perp$, so triangle is right. So Pythagorean Thm gives $|\vec{u}|^2+|\vec{v}|^2=|\vec{w}|^2$. Since $|\vec{u}|=|\vec{v}|=1$, $|\vec{w}|^2=1^2+1^2=2 \Rightarrow \boxed{|\vec{w}|=\sqrt{2}}$

12.2.34



$$\vec{w} = 50\sin 45^\circ\,\vec{i} - 50\cos 45^\circ\,\vec{j}$$
$$= 25\sqrt{2}\,\vec{i} - 25\sqrt{2}\,\vec{j}$$
$$\vec{p} = 250\sin 60^\circ\,\vec{i} + 250\cos 60^\circ\,\vec{j}$$
$$= 125\sqrt{3}\,\vec{i} + 125\,\vec{j}$$

Denote resultant by $\vec{r}$.

$$\vec{r} = \vec{w}+\vec{p} = 25\left[(\sqrt{2}+5\sqrt{3})\vec{i}+(-\sqrt{2}+5)\vec{j}\right]$$

true course $= \tan^{-1}\left(\dfrac{\sqrt{2}+5\sqrt{3}}{-\sqrt{2}+5}\right) \approx \boxed{N70.41^\circ E}$

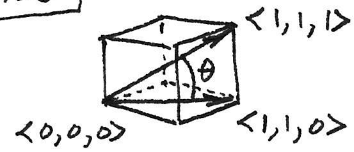
ground speed $= 25\sqrt{(\sqrt{2}+5\sqrt{3})^2+(-\sqrt{2}+5)^2} \approx \boxed{267.34 \text{ km/h}}$

12.3.42

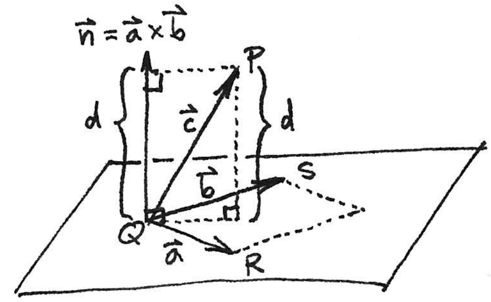
$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{\langle -2,3,-6\rangle\cdot\langle 5,-1,4\rangle}{|\langle -2,3,-6\rangle|} = \frac{-10-3-24}{\sqrt{(-2)^2+3^2+(-6)^2}} = \boxed{-\frac{37}{7}}$$

$$\text{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|} = \boxed{-\frac{37}{49}\langle -2,3,-6\rangle}$$

12.3.56. $\theta = \cos^{-1}\left(\dfrac{\langle 1,1,1\rangle \cdot \langle 1,1,0\rangle}{|\langle 1,1,1\rangle||\langle 1,1,0\rangle|}\right) = \cos^{-1}\left(\dfrac{2}{\sqrt{6}}\right) \approx \boxed{35.26^\circ}$



12.4.46 (a.) Let $\vec{n} = \vec{a}\times\vec{b}$. Distance d from pt P to plane that passes through Q, R, S is equal to magnitude of comp. of $\vec{c}$ in direction of $\vec{n}$:



$$\begin{aligned} d &= \left|\text{comp}_{\vec{n}}\,\vec{c}\right| \\ &= \left|\frac{\vec{n}\cdot\vec{c}}{|\vec{n}|}\right| && \text{def of scalar comp.} \\ &= \left|\frac{(\vec{a}\times\vec{b})\cdot\vec{c}}{|\vec{a}\times\vec{b}|}\right| && \text{def. of } \vec{n} \\ &= \frac{|\vec{a}\cdot(\vec{b}\times\vec{c})|}{|\vec{a}\times\vec{b}|} && \text{(5) of Thm 11} \end{aligned}$$

(b.) $\vec{a} = \overrightarrow{QR} = (0,2,0)-(1,0,0) = \langle -1,2,0\rangle$

$\vec{b} = \overrightarrow{QS} = (0,0,3)-(1,0,0) = \langle -1,0,3\rangle$

$\vec{c} = \overrightarrow{QP} = (2,1,4)-(1,0,0) = \langle 1,1,4\rangle$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 6\vec{i} + 3\vec{j} + 2\vec{k}$$

$$d = \frac{|\vec{a}\cdot(\vec{b}\times\vec{c})|}{|\vec{a}\times\vec{b}|} = \frac{|(\vec{a}\times\vec{b})\cdot\vec{c}|}{|\vec{a}\times\vec{b}|} = \frac{|\langle 6,3,2\rangle\cdot\langle 1,1,4\rangle|}{|\langle 6,3,2\rangle|} = \boxed{\frac{17}{7}}$$

12.4.53 (a.) No: $\vec{i}\cdot\vec{j} = \vec{i}\cdot\vec{k} = 0$ but $\vec{j} \neq \vec{k}$

(b.) No: $\vec{i}\times\vec{j} = \vec{i}\times(\vec{i}+\vec{j}) = \vec{k}$ but $\vec{j} \neq \vec{i}+\vec{j}$

(c.) Yes: $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} \Rightarrow \vec{a}\cdot(\vec{b}-\vec{c}) = 0 \Rightarrow |\vec{a}||\vec{b}-\vec{c}|\cos\theta = 0$

$\vec{a}\times\vec{b} = \vec{a}\times\vec{c} \Rightarrow \vec{a}\times(\vec{b}-\vec{c}) = 0 \Rightarrow |\vec{a}||\vec{b}-\vec{c}|\sin\theta = 0$

where θ is angle btwn $\vec{a}$ & $(\vec{b}-\vec{c})$. $\cos\theta$ & $\sin\theta$ cannot both be zero, & $\vec{a} \neq 0$ ($\Rightarrow |\vec{a}| = 0$), so $|\vec{b}-\vec{c}| = 0 \Rightarrow \vec{b} = \vec{c}$

D.P.3 WLOG, define $A = \frac{pq}{2}$, $B = \frac{pr}{2}$, $C = \frac{ab}{2}$, $D = \frac{bc}{2}$.

$$\begin{aligned} A^2+B^2+C^2 &= \tfrac{1}{4}(p^2q^2+p^2r^2+a^2b^2) \\ &= \tfrac{1}{4}(p^2b^2+a^2b^2) && \text{Pythagorean Thm} \\ &= \tfrac{1}{4}b^2c^2 && \text{Pythagorean Thm} \\ &= D^2 \end{aligned}$$