# HW 2 Solutions
# (Math 255 W13)

<u>12.5</u>

(22) $\bar{v}_1 = \langle 1,-1,3\rangle$ & $\bar{v}_2 = \langle 2,-2,7\rangle$ Not parallel

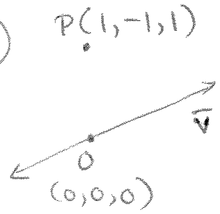
$L_1$ $\bar{r}_1(t) = \langle t, 1-t, 2+3t\rangle$

$L_2$ $\bar{r}_2(s) = \langle 2+2s, 3-2s, 7s\rangle$

$$\left.\begin{array}{l} t = 2+2s \\ 1-t = 3-2s \end{array}\right\} \Rightarrow -1-2s = 3-2s \quad \text{Not possible}$$

No point of intersec$^n$ $\Rightarrow$ Lines are skew

(36)




Line $x = 2y = 3z \iff \dfrac{x}{6} = \dfrac{y}{3} = \dfrac{z}{2}$

$\bar{v} = \langle 6,3,2\rangle$

$\overrightarrow{OP} = \langle 1,-1,1\rangle$ & $\bar{v}$ both lie along the plane

$\therefore\ \bar{n} \perp \overrightarrow{OP}$ & $\bar{n} \perp \bar{v}$

Choose $\bar{n} = \overrightarrow{OP} \times \bar{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 6 & 3 & 2 \end{vmatrix} = \langle -5, 4, 9\rangle$

Eq$^n$ of the plane is $-5x+4y+9z = 0$

(40) Plane 1 : $x - z = 1$ $\bar{n}_1 = \langle 1,0,-1\rangle$

Plane 2 : $y+2z = 3$ $\bar{n}_2 = \langle 0,1,2\rangle$

Plane 3 : $x+y-2z = 1$ $\bar{n}_3 = \langle 1,1,-2\rangle$

Plane 4 (the desired plane) $\perp$ Plane 3 $\Rightarrow \bar{n}_4 \perp \bar{n}_3$

Let $\bar{v}$ = line of $\cap^n$ of Planes 1 & 2

Plane 4 contains this line $\Rightarrow \bar{n}_4 \perp \bar{v}$

We can choose $\bar{n}_4 = \bar{n}_3 \times \bar{v}$

Now $\bar{v}$ lies along Planes 1 & 2 $\Rightarrow \bar{v} \perp \bar{n}_1$ & $\bar{v} \perp \bar{n}_2$

Let $\bar{v} = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = \langle 1,-2,1\rangle$

$$\bar{n}_4 = \begin{vmatrix} i & j & k \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix} = \langle -3, -3, -3 \rangle \text{ or } \langle 1, 1, 1 \rangle$$

Find a point on the line ($x - z = 1$ & $y + 2z = 3$) where $z = 0$ $\Rightarrow x = 1, y = 3$ $\therefore P(1, 3, 0)$

Eq$^n$ of Plane 4 is $1(x-1) + 1(y-3) + 1(z-0) = 0$

$$x + y + z = 4$$

(62) Let $A(2,5,5)$ & $B(-6,3,1)$

The plane passes through the midpoint of $AB$

$$= \left(\frac{2-6}{2}, \frac{5+3}{2}, \frac{5+1}{2}\right) = (-2, 4, 3)$$

& it is perpendicular to $AB$

$\therefore \bar{n} = \overrightarrow{AB} = \langle -8, -2, -4 \rangle$ or $\langle 4, 1, 2 \rangle$

Eq$^n$ of plane : $4(x+2) + 1(y-4) + 2(z-3) = 0$

$$4x + y + 2z = 2$$

12.6

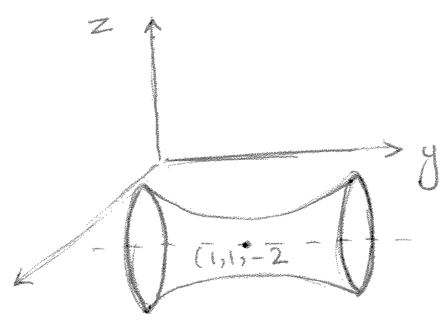
(36) $x^2 - 2x + 1 - (y^2 - 2y + 1) + (z^2 + 4z + 4) = -2 + 1 - 1 + 4$

$$(x-1)^2 - (y-1)^2 + (z+2)^2 = 2$$

Compare to $X^2 - Y^2 + Z^2 = 2$ or $Y^2 = X^2 + Z^2 - 2$

Hyperboloid of one sheet

Axis of symmetry // $y$-axis passing through $(1, 1, -2)$




(46) $P(x, y, z)$

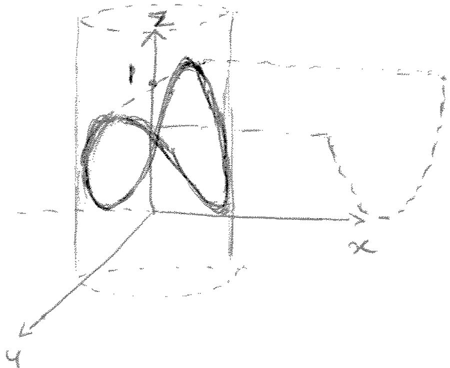
$\sqrt{y^2 + z^2} = 2|x| \Leftrightarrow y^2 + z^2 = 4x^2$

This is a cone with vertex at $(0,0,0)$ & axis of symmetry // $x$-axis

13.1

28 $\quad x^2+y^2 = \sin^2 t + \cos^2 t = 1$ ✓

$z = \sin^2 t = x^2$ ✓




48 $\quad$ ① $t = 1+2s$

② $t^2 = 1+6s$

③ $t^3 = 1+14s$

$$\left.\begin{array}{l} t = 1+2s \\ t^2 = 1+6s \end{array}\right\} \Rightarrow t^2-3t = 1+6s-3(1+2s) = -2$$

$$t^2-3t+2=0$$

$$t=1 \quad \text{or} \quad t=2$$

① $\Rightarrow s=0 \quad$ or $\quad s=\frac{1}{2}$

Check ③ $\quad 1 = 1+14(0)$ ✓ $\quad t=1,\ s=0$ works

$2^3 = 1+14\left(\frac{1}{2}\right)$ ✓ $\quad t=2,\ s=\frac{1}{2}$ works

Points of intersection are $(1,1,1)$ & $(2,4,8)$.

But $t$ & $s$ values are different at each point

$\Rightarrow$ no collision.