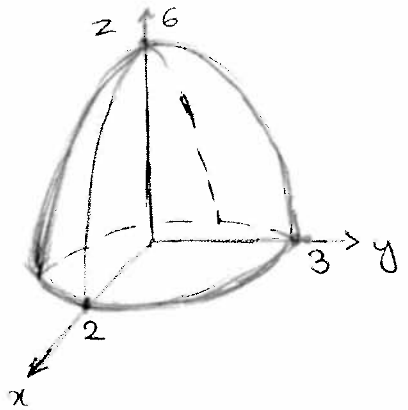
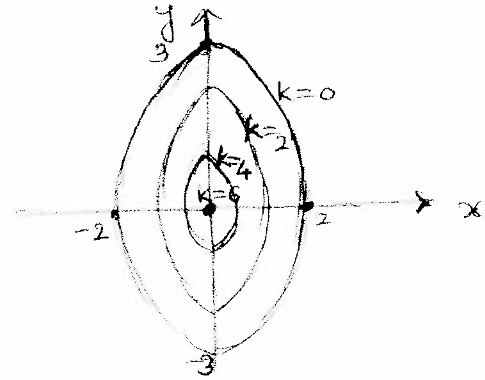
Math 255 – W13

HW 4 Solutions

14.1 52 $\quad z = \sqrt{36 - 9x^2 - 4y^2}$

$\Leftrightarrow \quad z \geq 0$ & $9x^2 + 4y^2 + z^2 = 36 \Leftrightarrow$ Top half of ellipsoid





14.2 16

$$0 \leq x^2 \leq x^2 + 2y^2$$

$$0 \leq \frac{x^2}{x^2+2y^2} \leq 1$$

$$0 \leq \frac{x^2 \sin^2 y}{x^2+2y^2} \leq \sin^2 y$$

$$\lim_{y \to 0} \sin^2 y = 0$$

Squeeze Th^m $\Rightarrow \displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{x^2+2y^2} = 0$

38 If $(x,y) \neq (0,0)$ $\quad f$ is a rational fn $\Rightarrow f$ is cont

$$\lim_{(x,0)\to(0,0)} \frac{xy}{x^2+xy+y^2} = \lim_{x\to 0} \frac{0}{x^2} = 0$$

$$\lim_{(x,x)\to(0,0)} \frac{xy}{x^2+xy+y^2} = \lim_{x\to 0} \frac{x^2}{3x^2} = \frac{1}{3}$$

2 different paths to $(0,0)$ yield 2 different limits

$\Rightarrow \displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ DNE.

$\therefore f$ is not cont at $(0,0)$.

14.3 72 Let $f(x,y,z) = \sqrt{1+xz}$ & $h(x,y,z) = \sqrt{1-xy}$

All partial derivatives of f & h are cont, if they exist

Clairaut's Th^m applies.

$$g = f + h \Rightarrow g_{xyz} = f_{xyz} + h_{xyz}$$
$$= f_{yxz} + h_{zxy}$$
$$= 0 \quad \text{since } f_y = 0 \ \& \ h_z = 0$$

100 $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$$= \lim_{h\to 0} \frac{\sqrt[3]{h} - 0}{h} = \lim_{h\to 0} \frac{h}{h}$$

$\Rightarrow f_x(0,0) = 1$

14.4 34 $h_0 = 10$ cm $\quad r_0 = 2$ cm

$\Delta h = 2(0.1) = 0.2$ cm $\quad \Delta r = 0.05$ cm

$V = \pi r^2 h \Rightarrow V_r = 2\pi r h$, $V_h = \pi r^2$

Amount of metal $= \Delta V \approx V_r(r_0,h_0)\Delta r + V_h(r_0,h_0)\Delta h$

$$= 40\pi(0.05) + 4\pi(0.2)$$
$$= 2.8\pi \text{ cm}^3$$

42 $\bar{r}_1(0) = \langle 2,1,3\rangle$ & $\bar{r}_2(1) = \langle 2,1,3\rangle$

Tangent vectors $\bar{r}_1'(0)$ & $\bar{r}_2'(1)$ must lie in the tangent plane at P.

$\therefore \bar{n} = \bar{r}_1'(0) \times \bar{r}_2'(1)$

$\bar{r}_1'(t) = \langle 3, -2t, -4+2t\rangle$

$\bar{r}_1'(0) = \langle 3, 0, -4\rangle$

$\bar{r}_2'(u) = \langle 2u, 6u^2, 2\rangle$

$\bar{r}_2'(1) = \langle 2, 6, 2\rangle$

$$\bar{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -4 \\ 2 & 6 & 2 \end{vmatrix} = \langle 24, -14, 18 \rangle$$

or $\langle 12, -7, 9 \rangle$

Eq^n is $12(x-2) - 7(y-1) + 9(z-3) = 0$

$$12x - 7y + 9z = 44$$

46 (a) $f_y(0,0) = \lim\limits_{h \to 0} \dfrac{f(0,h) - 0}{h} = \lim\limits_{h \to 0} \dfrac{0}{h} = 0$

Similarly $f_x(0,0) = 0$

$$\lim_{(x,0) \to (0,0)} \frac{xy}{x^2+y^2} = \lim_{x \to 0} \frac{0}{x^2} = 0$$

$$\lim_{(x,x) \to (0,0)} \frac{xy}{x^2+y^2} = \lim_{x \to 0} \frac{x^2}{2x^2} = \frac{1}{2}$$

2 different paths, 2 different limits

$\Rightarrow \lim\limits_{(x,y) \to (0,0)} \dfrac{xy}{x^2+y^2}$ DNE

$\Rightarrow$ f : not cont at $(0,0)$

\# 45 $\Rightarrow$ f : not differentiable at $(0,0)$.

(b) $f_x = \dfrac{y(x^2+y^2) - xy(2x)}{(x^2+y^2)^2} = \dfrac{y(y^2-x^2)}{(x^2+y^2)^2}$ if $(x,y) \neq (0,0)$

$f_y = \dfrac{x(x^2+y^2) - xy(2y)}{(x^2+y^2)^2} = \dfrac{x(x^2-y^2)}{(x^2+y^2)^2}$ if $(x,y) \neq (0,0)$

So f_x, f_y exist on $\mathbb{R}^2$

If f_x & f_y were also cont at $(0,0)$

then Th^m 8 $\Rightarrow$ f : diffble at $(0,0)$,

which is not true as seen in (a)

$\therefore$ f_x or f_y not cont at $(0,0)$

By symmetry, f_x & f_y must both be discont at $(0,0)$.

<u>OR</u>

Along $x=0$ $\quad f_x(0,y) = \dfrac{y^3}{y^4} = \dfrac{1}{y}$

$\displaystyle\lim_{y\to 0} f_x(0,y)$ DNE

$\therefore \displaystyle\lim_{(x,y)\to(0,0)} f_x(x,y)$ DNE $\Rightarrow f_x$ not cont at $(0,0)$

Similarly f_y not cont at $(0,0)$
(Use x-axis)