Math 255-W13

HW 6 Solutions

15.2 (8)

$$\int_1^3\int_1^5 \frac{\ln y}{xy}\,dy\,dx = \int_1^5 (\ln y)\left(\tfrac{1}{y}\,dy\right) \times \int_1^3 \frac{1}{x}\,dx$$

$$= \frac{(\ln y)^2}{2}\Big|_1^5 \times \ln|x|\Big|_1^3$$

$$= \frac{1}{2}(\ln 5)^2 \ln 3$$

(20) $\displaystyle\iint_R \frac{x}{1+xy}\,dA \qquad R = [0,1]\times[0,1]$

$$= \int_0^1\int_0^1 \frac{x}{1+xy}\,dy\,dx \qquad (u = 1+xy \Rightarrow du = x\,dy)$$

$$= \int_0^1 \ln(1+xy)\Big|_0^1 dx$$

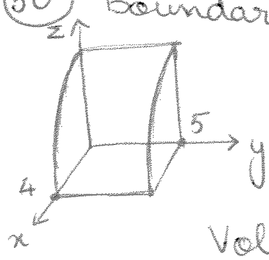
$$= \int_0^1 \underbrace{\ln(1+x)}_{u}\,\underbrace{dx}_{dv} = x\ln(1+x)\Big|_0^1 - \int_0^1 \frac{x}{1+x}\,dx$$

$$= \ln 2 + \int_0^1\left(\frac{1}{1+x} - 1\right)dx$$

$$= \ln 2 + \Big[\ln(1+x) - x\Big]_0^1$$

$$= 2\ln 2 - 1$$

(30) Boundaries $x=0,\ y=0,\ z=0,\ z=16-x^2,\ y=5$



$0 \le z \le 16-x^2 \qquad 16-x^2 = 0 \Rightarrow x = \pm 4$

$\therefore$ Base in x-y plane $= [0,4]\times[0,5]$

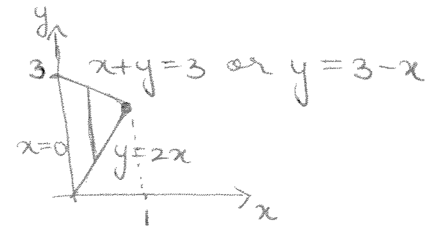
$$\text{Volume} = \int_0^4\int_0^5 (16-x^2)\,dy\,dx$$

$$= \int_0^5 dy \times \int_0^4 (16-x^2)\,dx$$

$$= 5\left[16x - \frac{x^3}{3}\right]_0^4$$

$$= 5\left[64 - \frac{64}{3}\right] = \frac{640}{3}$$

15.3 (22) $\displaystyle\iint_D 2xy\,dA$



$$= \int_0^1 \int_{2x}^{3-x} 2xy\,dy\,dx$$

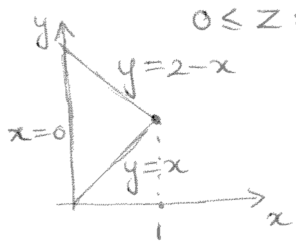
$$= \int_0^1 xy^2 \Big|_{2x}^{3-x} dx = \int_0^1 x(9-6x+x^2-4x^2)\,dx$$

$$= \int_0^1 (9x-6x^2-3x^3)\,dx$$

$$= \frac{9x^2}{2} - 2x^3 - \frac{3}{4}x^4 \Big|_0^1 = \frac{9}{2} - 2 - \frac{3}{4} = \frac{7}{4}$$

(28) Boundaries $z=x$, $z=0$, $y=x$, $x+y=2$

$0 \le z \le x$ Base in xy plane bounded by $y=x$, $x+y=2$ and $x=0$ $(z=0 \cap z=x)$



$$\text{Vol} = \int_0^1 \int_x^{2-x} x\,dy\,dx = \int_0^1 x(2-2x)\,dx$$

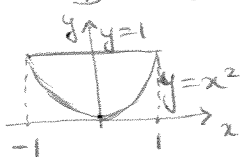
$$= \int_0^1 (2x-2x^2)\,dx$$

$$= x^2 - \frac{2}{3}x^3 \Big|_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$$

(36) Boundaries $y=x^2$, $z=3y$, $z=2+y$

$3y \le z \le 2+y$ over D in xy plane

D: bounded by $y=x^2$ & $y=1$ $(z=3y \cap z=2+y$
$\Rightarrow 3y = 2+y \Rightarrow 2y=2 \Rightarrow y=1)$



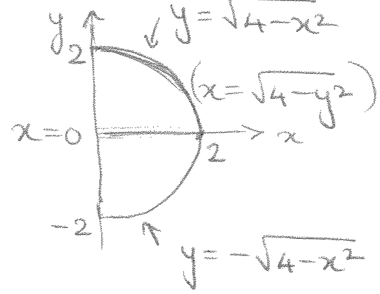
$$\text{Volume} = \int_{-1}^1 \int_{x^2}^1 (2+y-3y)\,dy\,dx$$

$$= \int_{-1}^1 \left[2y - y^2\right]_{x^2}^1 dx$$

$$= \int_{-1}^1 (1-2x^2+x^4)\,dx = x - \frac{2}{3}x^3 + \frac{x^5}{5} \Big|_{-1}^1$$

$$= 2 - \frac{4}{3} + \frac{2}{5} = \frac{16}{15}$$

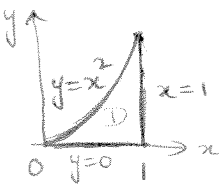
(46) $\begin{cases} 0 \le x \le \sqrt{4-y^2} \\ -2 \le y \le 2 \end{cases}$



$$\Longleftrightarrow \begin{cases} -\sqrt{4-x^2} \le y \le \sqrt{4-x^2} \\ 0 \le x \le 2 \end{cases}$$

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}} f(x,y)\,dx\,dy = \int_{0}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y)\,dy\,dx$$

(60)



$$\iint_D x \sin y \, dA = \int_0^1 \int_0^{x^2} x \sin y \, dy\, dx$$

$$= -\int_0^1 x \cos y \Big|_0^{x^2} dx$$

$$= -\int_0^1 (x \cos(x^2) - x)\, dx \qquad \left(\begin{array}{l} u = x^2 \\ \frac{1}{2} du = x\,dx \end{array} \right)$$

$$= -\frac{1}{2}\sin(x^2) + \frac{x^2}{2} \Big|_0^1$$

$$= \frac{1-\sin 1}{2}$$

$$\text{Area of } D = \iint_D dA = \int_0^1 \int_0^{x^2} dy\,dx$$

$$= \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$f_{ave} \text{ over } D = \frac{\iint_D x \sin y\, dA}{\text{Area of } D} = \frac{3}{2}(1-\sin 1)$$